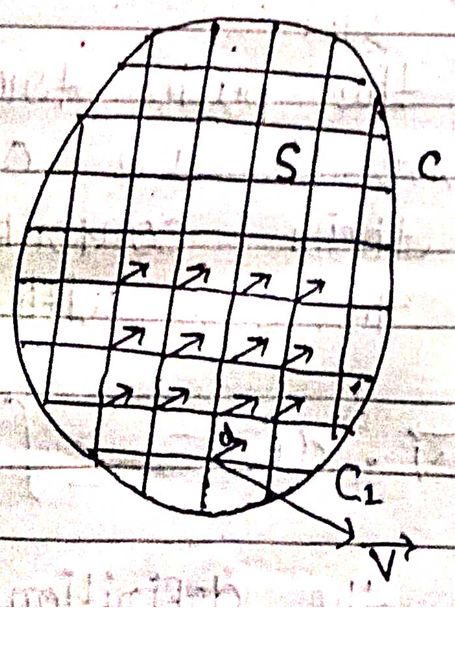
* Stoke's Theorem :
This theorem states that the line intergral of a vector $\vec{V}$ around any closed curve C is equal to the surface intergal of curl $\vec{V}$ taken over any surface S of which the curve C is a boundary edge. i.e.,

$$\oint_c \vec{V} \cdot d\vec{l} = \iint \text{curl } \vec{V} \cdot d\vec{S} = \iint_S (\nabla \times \vec{V}) \cdot d\vec{S}$$

Proof:
Let a vector $\vec{V}$ be a function of position. Then its line intergal along a closed curve C is given by

$$L = \oint_C \vec{V} \cdot d\vec{l},$$



Where $d\vec{l}$ is a small element of path as shown in fig. Let us divide the enclosed area by drawing lines over it into a large number of small areas $dS_1, dS_2 \ldots\ldots dS_n$ bounded by the curves $C_1, C_2, C_3 \ldots\ldots C_n$.

Let us suppose that the curve C_1 is divided into two parts to form two separate closed curves C_1' and C_1''. If L_1 and L_2 are the line intergrals from the two parts of the curve under consideration, then

$$L = L_1 + L_2$$

$$\text{or} \quad \oint_C \vec{V} \cdot d\vec{l} = \oint_{C_1'} \vec{V} \cdot d\vec{l} + \oint_{C_1''} \vec{V} \cdot d\vec{l}.$$

Since the paths traversed by the two of the curve C_1 are oppositely traced and hence their contribution is cancelled. Similar is the case with other curves.

$$\therefore \iint_C \vec{V} \cdot d\vec{l} = \sum \iint_{C_n} \vec{V} \cdot dl.$$

But from the definition of curl $\vec{V}$, we have

$$\oint_C \vec{V} \cdot d\vec{l} = \text{Curl } \vec{V} \cdot d\vec{S_n}$$

Hence the surface area is $d\vec{S_n}$, so we have

$$\text{Curl } \vec{V} \cdot d\vec{S_n} = \oint_{C_n} \vec{V} \cdot d\vec{l}$$

$$\therefore \oint \vec{V} \cdot d\vec{l} = \sum \oint_{C_n} \vec{V} \cdot dl = \sum \text{Curl } \vec{V} \cdot d\vec{S_n}$$

The summation on the R.H.S. is clearly the surface integral of Curl $\vec{V}$ over the entire surface S enclosed by the curve so that we have,

$$\oint_C \vec{V} \cdot d\vec{l} = \iint \text{Curl } \vec{V} \cdot d\vec{S}$$

$$= \iint_S (\nabla \times \vec{V}) \cdot d\vec{S}$$